Impact-Generated Atmospheres over Titan, Ganymede, and Callisto

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The competition between impact erosion and impact supply of volatiles to planetary atmospheres can determine whether a planet or satellite accumulates an atmosphere. In the absence of other processes (e.g., outgassing), we find either that a planetary atmosphere should be thick, or that there should be no atmosphere at all. The boundary between the two extreme cases is set by the mass and velocity distributions and intrinsic volatile content of the impactors. We apply our model specifically to Titan, Callisto, and Ganymede. The impacting population is identified with comets, either in the form of stray Uranus-Neptune planetesimals or as dislodged Kuiper belt comets. Systematically lower impact velocities on Titan allow it to retain a thick atmosphere, while Callisto and Ganymede get nothing. Titan's atmosphere may therefore be an expression of a late-accreting, volatile-rich veneer. An impact origin for Titan's atmosphere naturally accounts for the high D/H ratio it shares with Earth, the carbonaceous meteorites, and Halley. It also accounts for the general similarity of Titan's atmosphere to those of Triton and Pluto, which is otherwise puzzling in view of the radically different histories and bulk compositions of these objects. © 1992 Academic Press, Inc.

1. INTRODUCTION

Ganymede, Callisto, and Titan are of similar size, mass, and density, and so presumably share similar bulk composition. Aside from their belonging to different planets, there is only one obvious difference between Titan on the one hand and Ganymede and Callisto on the other: Titan has a thick atmosphere, and Callisto and Ganymede have nothing. We will argue here that this sharp contrast is a predictable consequence of impact erosion, i.e., the escape of atmospheric gases as a consequence of hypervelocity impacts. From the standpoint of their ability to withstand impact erosion, the essential difference between the three satellites is the distribution of impact velocities of incident material. Because Saturn is less massive than Jupiter, and because Jupiter sits deeper in the Sun's gravitational well, the average impact velocity of stray bodies striking Titan is lower than those striking Callisto or Ganymede. Other things being equal, the lower impact velocity allows Titan to retain a higher fraction of incoming atmophiles, and to suffer less atmospheric erosion, than its Jovian counterparts. The question we will want to address here is whether this difference alone could suffice to account for their different fates.

2. ATMOSPHERIC CRATERING

Atmospheric cratering (a.k.a. impact erosion) refers to the expulsion of atmospheric gases by impact. It has been suggested, in diverse forms and various contexts, as an important loss process for planetary volatiles from the terrestrial planets (Cameron 1983, Watkins 1983, Walker 1986, Ahrens and O'Keefe 1987, Zahnle et al. 1988, Ahrens et al. 1989, Hunten et al. 1989, Melosh and Vickery, 1989, Chyba 1990). Three more-or-less orthogonal detailed mechanisms for atmospheric cratering have been developed. Walker (1986) emphasized the interaction of the impactor with the air it encounters on its way to the surface. Escape was limited to the air directly encountered by the impactor, like a cookie cutter. Ahrens and O'Keefe (1987) treated the impact as a massless point explosion at the base of an exponential atmosphere. They predicted somewhat more escape than Walker. The most effective plausible mechanism for expelling atmosphere in an impact was proposed by Melosh and Vickery (1989). These authors considered both the velocity and the momentum of the ejecta, and concluded that for sufficiently large, high speed impacts the ejecta could snowplow all the atmosphere above the horizon into space. Where impact velocities are generally much higher than the escape velocity, the snowplow model is the most effective of the three. It should dominate impact erosion from the large icy satellites.

In Melosh and Vickery's model atmospheric gases are swept to space by the momentum of a hydrodynamically expanding cloud of superheated vapor (and particles that condense from it) produced by impact. Escape occurs if there is enough extra momentum in the potentially escaping ejecta to accelerate the intervening atmosphere to escape velocity. For the present purpose errors of a factor of a few are of no moment, provided that the description of atmospheric cratering retains the important qualitative features. Thus for a first cut we will use the "tangentplane" parameterization suggested by Melosh and Vickery (1989). This assumes that when atmospheric cratering occurs, all the atmosphere above the horizon is expelled; i.e., that part of the atmosphere lying above a plane tangent to the surface of the planet or satellite¹ escapes. Escape occurs if (i) the expansion velocity of the rock vapor exceeds the escape velocity from the planet, and (ii) the mass of rock vapor exceeds the mass of atmosphere above the tangent-plane.²

The first criterion is deemed satisfied if the average thermal velocity of the rock vapor exceeds the escape velocity. Melosh and Vickery assume that all the internal energy initially deposited by the shock is left where it was put as heat in the shocked material; i.e., they assume that the shock is perfectly inelastic. They subtract from this the latent heat of vaporization to obtain the thermal energy available to power expansion of the vapor. With these assumptions Melosh and Vickery obtained v_c as a lower bound on the impact velocity required for large-scale escape

$$v \ge v_c \equiv 2\sqrt{v_{\rm esc}^2 + 2L_{\rm vap}},\tag{1}$$

where $v_{\rm esc}$ is the escape velocity and $L_{\rm vap}$ is the latent heat of vaporization (for which Melosh and Vickery use 1.3×10^{11} erg/g for rock, and 3×10^{10} erg/g for ice). This expression assumes the impact of identical materials; a somewhat more complicated expression is needed for impact of rock and ice. Since it overestimates the amount of heat retained in the shocked material, Eq. (1) underestimates v_c . Later, in Section 5.5, we will modify Eq. (1) to allow for some elasticity in the shocked material.

The second criterion is deemed satisfied if the mass of

the impactor exceeds the mass of atmosphere above the horizon. Melosh and Vickery then presume that there is enough momentum in the rock vapor to blow off all the atmosphere encountered. The fraction of an exponential atmosphere above a plane tangent to the surface is H/2R, where H is the density scale height and R the planet's radius. This expression is accurate to order H/R. The minimum impactor mass for escape can then be written

$$m \ge m_c \equiv \frac{H}{2R} M Y_{\rm a}, \qquad (2)$$

where Y_a stands for the total mass of the atmosphere normalized to the mass M of the planet (grams of atmosphere per gram of planet). Equation (2) is, for the atmosphere, potentially disastrous. It states that a thinner atmosphere can be eroded by smaller impactors. Since smaller impactors are more numerous, thinner atmospheres erode more rapidly. Thus once an atmosphere begins to erode, impact erosion accelerates until the planet is wholly stripped (Melosh and Vickery 1989).

3. ATMOSPHERIC EVOLUTION BY IMPACT

This study begins with the premise that early atmospheric evolution was driven by impacts. Impactors will be viewed as the sole source and atmospheric cratering the sole sink of the atmosphere. Emphasis is placed on the accretion of a late, volatile-rich veneer. Other processes—e.g., hydrodynamic escape, outgassing, and volatile recycling—are all neglected, not because they are negligible (they are not), but in order to focus attention on what atmospheric cratering can and cannot do. Aside from relative simplicity, the key advantage of neglecting these other processes is that the impact flux can be discussed in terms of the total mass accreted rather than in terms of an ill-known if not wholly arbitrary explicit function of time.

Consider the competition between volatile accretion (e.g., impact degassing) and impact erosion of an atmosphere. Let y_j denote the abundance of an atmospheric species *j* in grams per gram of impacting material, and let Y_j denote the abundance of *j* in grams per gram of planet. The species *j* will be considered an obligate atmophile; i.e., a volatile with no significant reservoir other than the atmosphere. The planetary inventory of *j* then evolves according to the difference between its delivery in some impacts and its expulsion by others. This competition can be represented by the following equation, in which the planet of mass *M* accretes at the rate *M*:

$$\dot{Y}_j = \frac{\dot{M}}{M} (\chi y_j - \eta Y_j - Y_j).$$
(3)

¹ We will generally use the concrete if not always strictly appropriate "planet" rather than "object" or "body" when referring in general to a planet or large moon.

 $^{^{2}}$ More detailed models of the phenomenon that account for different slant paths through the atmosphere and some other relevant matters are discussed by Vickery and Melosh (1990) and Zahnle (1990).

The first term on the rhs of Eq. (3) is the contribution of the impactor's atmophiles to the atmosphere; the factor χ $(0 \le \chi \le 1)$ represents the fraction of the impactor's volatiles released into the atmosphere and retained by the planet. The second term represents atmospheric cratering. The cumulative effects of atmospheric cratering are integrated into a single nondimensional factor η , which we have defined to be the ratio of the planet's accretion time scale to the atmosphere's escape time scale; when escape is important η is large. The third term accounts for the growing mass of the planet. It is negligible if impact erosion is important (i.e., if $\eta \ge 1$) or if one is considering only a late-accreting veneer of volatile-rich material.³ Neither the accretion efficiency χ nor the atmospheric cratering η would be expected to discriminate among obligate atmophiles, and so neither has been written with the subscript j. However, in the general case χ and η would be functions of *j*.

3.1. χ and η

The source and sink functions χ and η are average values determined by integrating the effects of individual impacts over the population of impactors. We will assume that y_j is constant; i.e., that the composition of the impactors is on average independent of impact velocity and mass. An average source efficiency χ can be defined by

$$\chi = \frac{1}{\dot{M}} \iint f_{\chi}(m, v) \, \dot{n}(m, v) \, m \, dm \, dv, \qquad (4)$$

where $f_x(m, v)$ refers to the fraction of a given impactor's volatile inventory retained by the planet. The differential impactor flux as a function of mass and velocity is denoted $\dot{n}(m, v)$. The mass accretion rate \dot{M} is

$$\dot{M} = \iint \dot{n}(m, v) \, m \, dm \, dv. \tag{5}$$

Equation (4) can easily be generalized to include other factors not addressed here.

The atmospheric cratering efficiency can be defined in terms of Y_j , but η is more naturally expressed in terms of the atmospheric inventory N_j (the total number of atoms of j in the atmosphere):

$$\eta \equiv \frac{|\tau_{\rm acc}|}{|\tau_{\rm esc}|} = -\frac{N_j}{N_j} \frac{M}{\dot{M}}.$$
 (6)

³ Equation (3) is not entirely self-consistent. With impact erosion \dot{M} should be identified not with the mass accreted by the planet but instead with the mass incident on the planet. The third term in Eq. (3) would then need to be multiplied by the ratio of mass accreted to mass incident. In practice, for thin veneers the third term is negligible, so the issue does not arise.

 η can then be evaluated in terms of the fraction $\xi(m, v)$ of the planet's volatile inventory that escapes in an impact,

$$\eta = \frac{M}{\dot{M}} \iint \xi(m, v) \, \dot{n}(m, v) \, dm \, dv. \tag{7}$$

For an obligate atmophile, like a noble gas, the fraction ξ is equal to the fraction of the atmosphere expelled in an impact, while for a condensed volatile, like water on Earth, ξ would refer to the fraction of the ocean expelled. Since condensed volatiles are more likely to be retained, atmospheric cratering discriminates between strict atmophiles like the noble gases and probably nitrogen on the one hand, and water and possibly carbon dioxide on the other.

3.2. Mass Distribution of Impactors

Evaluation of χ and η requires a description of the mass and velocity distribution of the impactors. In this paper we will assume that the velocity distribution is independent of mass. This is equivalent to assuming that big and small impactors revolve in similar orbits. The impact flux can then be written as a product of separate mass and velocity distributions, $\dot{n}(m, v) dm dv = f(v) dv \dot{n}(m) dm$. Possible velocity distributions for interesting classes of impactors are discussed in Section 5.4.

It is usual and convenient to describe the mass spectrum of impactors by a power law

$$\dot{n}(m)\,dm = Cm^{-q}\,dm.\tag{8}$$

The appropriate value of the power law exponent q is not known. Expectations for q seem to range from $3/2 \leq q \leq$ 2, with the higher values corresponding to more violent mutual collisions among the objects. Dohnanyi (1972) and Safronov et al. (1986) favor q = 11/6 for a distribution dominated by fragmenting collisions. Melosh and Vickery (1989) and Chyba (1990) use q = 1.47 for the late heavy bombardment. This value is derived from lunar cratering statistics using the popular Schmidt-Housen (1987) crater scaling relationship. Large asteroids have $q \approx 2$ (Hughes 1982, Donnison and Sugden 1984). Because it is difficult to relate the masses and luminosities of comets, the cometary mass distribution is harder to estimate. Nor is it clear that the same power law applies to large and small comets (for more on this view, see Weissman 1990). Given these caveats, for long period comets q has been estimated as ~ 1.7 (Hughes 1988, Donnison 1986). For short period comets q has been estimated as 1.45 (Donnison 1986). As much as possible we will treat q as a free parameter, but we will often use q = 3/2 because it allows closed-form analytical solutions for many interesting cases, as well as direct comparison with previous work using q = 1.47.

The constant C is determined by integrating over the whole mass distribution to get the mass accretion rate \dot{M} . For q < 2.

$$\dot{M} = \int_0^\infty f(v) \, dv \int_0^{m_1} \dot{n}(m)m \, dm = \frac{m_1^{2-q}}{2-q} C. \tag{9}$$

The velocity distribution f(v) is normalized such that

$$\int_0^\infty f(v) \, dv = 1. \tag{10}$$

The resulting mass distribution is therefore

$$\dot{n}(m) dm = \frac{(2-q)\dot{M}}{m_1^{2-q}} m^{-q} dm.$$
 (11)

The largest impactor is denoted m_1 . Its mass in a power law distribution of impactors is statistically related to the total mass of all the impactors. Since we will mainly be discussing late-accreting volatile-rich veneers, it is useful to define the mass of veneer accreted after time t as

$$\delta M(t) \equiv \int_{t}^{\text{present}} \dot{M}(t) \, dt. \tag{12}$$

It should be noted that in models with impact erosion, the actual mass accreted is less than the incident mass δM . Indeed, it is entirely possible for a planet or moon to shrink under a high velocity bombardment. So long as we confine our attention to thin veneers ($\delta M \ll M$), there is no cause for confusion. The mass of the largest impactor m_1 in the veneer is statistically related to δM (Wetherill 1975) by

$$m_1 \approx \left(\frac{4-2q}{3-q}\right) \delta M. \tag{13}$$

It is important that m_1 scales with the mass of the veneer. If this is not taken into account (if, say, m_1 were treated as a constant), there would be far too many impacts if m_1 were too small, and major contributions from fictitious "fractional impacts" if m_1 were too large. To estimate m_1 the entire incident mass δM is used.

3.3 The Functions χ and η for Impact Erosion Following MV

In their study, Melosh and Vickery ignored any volatiles that might have been present in the impactors. However, as they point out elsewhere (Vickery and Melosh (1990), the impactor itself is the likeliest fraction of the ejecta to escape, since it is shocked early and to the greatest extent, and it is initially positioned toward the outside of the plume. Thus it is implicit in their model that when atmospheric cratering occurs the impactor is also lost. The natural extension of the Melosh—Vickery model is therefore to assume that if $v > v_c$ and $m > m_c$, all the volatiles in the impactor escape; otherwise they are retained. This means that (i) low velocity impacts and (ii) small objects contribute their volatiles with 100% efficiency to the atmosphere. Therefore

$$f_{\chi}(m, v) = \begin{cases} 0 & m > m_c \text{ and } v > v_c \\ 1 & m < m_c \text{ or } v < v_c. \end{cases}$$
(14)

It is useful to reduce the integral over velocity to a single parameter. Define F_{χ} to be the fraction of impactors with impact velocity less than v_c ; i.e.,

$$F_{\chi} \equiv \int_{0}^{\infty} f_{\chi}(m, v) f(v) \, dv = \int_{0}^{v_{c}} f(v) \, dv.$$
(15)

Along with y_j and q, F_{χ} (and its close relative, F_{η} , which will be defined shortly) is one of the three key free parameters in this study. F_{χ} can be regarded as the fraction of all impacts that arrive slowly enough to be retained. A more general definition of F_{χ} would allow for nonseparable velocity and mass integrals, as would arise if the effective value of v_c were a function of m. In our extension of the Melosh–Vickery approximation, F_{χ} is a measure of impact velocity vs escape velocity. Then for q < 2, integration of Eq. (4) yields

$$\chi = F_{\chi} + (1 - F_{\chi}) \left(\frac{H}{2R} \frac{M}{m_1} Y_{\rm a}\right)^{2-q}.$$
 (16)

The first term is the fraction of impactors that hit slowly enough that their volatile cargo is delivered successfully. The second term refers to those impactors with high velocities that are too small for their ejecta to escape—either they are stopped in the atmosphere, or their ejecta are smothered by the atmosphere.

The assumption that all the impactor's volatiles escape when atmospheric erosion occurs is not beyond question. These volatiles are released on impact, initially mixed with the rock vapor, and thus may be no more likely to escape than is the rock vapor itself. In the context of the simple Melosh–Vickery prescription this is self-consistent, since when atmospheric cratering occurs the impactor itself is implicitly assumed to escape. But in a more realistic description of the process only a fraction of a large impactor need escape to clear away the atmosphere above the tangent-plane. Indeed, very large impactors (with radii comparable to that of the target) potentially could remove considerably more than just the tangent plane—however, neither of these omissions is likely to be important, since large impactors are too few to contribute significantly to impact erosion. For almost all impacts escape is limited to the atmosphere above the tangentplane; hence impact erosion is dominated by relatively frequent impacts with mass of order m_c . Some consequences of the extreme alternative assumption, that none of the impactor's volatiles escape on impact, are considered in Section 5.6.

The function $\xi(m, v)$, the fraction of the atmosphere expelled by an impact, can be written in the Melosh-Vickery approximation as

$$\xi(m, v) = \begin{cases} \frac{H}{2R} \equiv \xi_0 & m > m_c \text{ and } v > v_c \\ m < m_c \text{ or } v < v_c. \end{cases}$$
(17)

Again, it is useful to represent the integral over velocity by a single parameter:

$$F_{\eta} = \frac{1}{\xi_0} \int_0^\infty \xi(m, v) f(v) \, dv = \int_{v_c}^\infty f(v) \, dv.$$
(18)

Because we have assumed that $\chi(v)$ and $\xi(v)$ are both step functions with the step at the same v_c , we have $F_{\eta} = 1 - F_{\chi}$. Hence introducing F_{η} has not necessarily introduced another free parameter. However, for the present we will carry both F_{χ} and F_{η} . Keeping them both is a convenient bookkeeping device for tracking the χ and η terms through future equations; they can also be varied separately to test the relative importance of supply and escape on the stability of atmospheres. When ξ is integrated over the distribution of impactors, Eq. (7) gives

$$\eta = F_{\eta} \frac{2-q}{q-1} \left(\xi_0 \frac{M}{m_1} \right)^{2-q} Y_a^{1-q}.$$
(19)

The above expression assumes that $(m_1/m_c)^{q-1} \ge 1$, i.e., that the largest impactor is much larger than the smallest impactor that can effect atmospheric erosion.

Although the tangent-plane prescription is crude, it should be emphasized that the important quantities, η and χ , are integrals taken over the mass and velocity distribution of impactors. Unless this distribution is pathological (if, say, it were distinguished by a peculiar preponderance of objects of mass m_c), a more sophisticated parameterization changes η and χ by no more than a factor of 2 or 3.

3.4.

With the above expressions for χ and η , Eq. (3) for the inventory of the atmophile *j* based on the tangent-plane model for atmospheric cratering becomes

$$\dot{Y}_{j} = \frac{\dot{M}}{M} \left\{ F_{\chi} y_{j} + (1 - F_{\chi}) y_{j} \left(\frac{m_{c}}{m_{1}} \right)^{2-q} - F_{\eta} \frac{2-q}{q-1} \frac{Y_{j}}{Y_{a}} \left(\frac{m_{c}}{m_{1}} \right)^{2-q} - Y_{j} \right\}.$$
 (20)

According to Eq. (13) the mass of the largest impactor m_1 is linearly proportional to the veneer mass δM . The atmospheric evolution equation (20) then becomes a first-order differential equation for y_j as a function of the veneer mass δM . It is convenient to define a nondimensional veneer mass

$$x \equiv \frac{\delta M}{M}.$$
 (21)

In the present epoch, which is marked by occasional impacts of order 10^{18} g, x is of the order 10^{-8} or 10^{-9} . Recall that $\delta M(t)$ has been defined as the veneer not yet accreted (see Eq. (12)), so that $x(t) \rightarrow 0$ as $t \rightarrow \infty$. This was done so that the mass of the largest impactor $m_1(t)$ could be related to x(t) in a relatively simple way. If the veneer is assumed thin we can treat M as constant. It is consistent with the approximation of a thin veneer to drop the last term on the right-hand side of equation (20) (the term involving Y_j alone), since this term arises from the changing mass of the planet. (This term could also be dropped simply on the grounds that it is necessarily small if atmospheric cratering is important.) With the above adjustments, when fully expanded Eq. (20) becomes

$$\frac{dY_j}{dx} = -F_{\chi}y_j - (1 - F_{\chi}) \left[\xi_0 \left(\frac{3 - q}{4 - 2q} \right) \right]^{2-q} Y_a^{2-q} y_j x^{q-2} + F_{\eta} \frac{2 - q}{q - 1} \left[\xi_0 \left(\frac{3 - q}{4 - 2q} \right) \right]^{2-q} Y_a^{1-q} Y_j x^{q-2}.$$
 (22)

The three terms on the rhs represent, respectively, (i) the volatiles contributed by slow impactors, (ii) the volatiles contributed by small, fast impactors, and (iii) impact erosion by large, fast impactors. Analytical solutions to Eq. (22) can be obtained given either a simple enough prescription for the evolving mass of the background atmosphere Y_a or for the mixing ratio Y_j/Y_a , or if the governing equations are further simplified. Examples of the former are to assume constancy of Y_a or Y_j/Y_a . An example of the latter is to assume that all impacts with $m > m_c$ are erosive, i.e., to take $F_{\chi} = 0$. This assumption is useful for estimating the maximum atmosphere that can be completely expelled by a given mass veneer. It is also useful for direct comparison with the work of Melosh and Vickery.

4. ATMOSPHERE EVOLUTION WITH CONSTANT MIXING RATIOS

Consider an atmosphere that has the same composition as the volatiles in the impactors. This is equivalent to taking Y_i/Y_a constant. Constant Y_i/Y_a means that the constituent j is always present in the atmosphere in constant mixing ratio. A trivial example is the one-volatile planet, for which $Y_i/Y_a = 1$. This case, which we will emphasize in this study, is useful for comparison with Melosh and Vickery (1989) and Chyba (1990). More generally, the assumption of constant Y_i/Y_a should hold for all volatiles if the atmosphere is their primary reservoir, since to first approximation atmospheric cratering does not discriminate between atmospheric constituents in a well-mixed atmosphere-either a moelcule is above the horizon or it is not. However, it would not hold if the major atmospheric constituent had its primary reservoir elsewhere and exchange was rapid. Such, for example, might be the case for CO₂ on Mars, if most Martian CO₂ is bound up in carbonates and recycling were efficient.

Given constant mixing ratios, Eq. (22) for q < 2 can be written

$$\frac{dY_j}{dx} = A_j Y_j^{2-q} x^{q-2} - F_{\chi} y_j,$$
(23)

where the constant

$$A_{j} = \left\{ \xi_{0} \left(\frac{3-q}{4-2q} \right) \right\}^{2-q} \left\{ F_{\eta} \frac{2-q}{q-1} \left(\frac{Y_{a}}{Y_{j}} \right)^{1-q} - (1-F_{\chi}) y_{j} \left(\frac{Y_{a}}{Y_{j}} \right)^{2-q} \right\}$$
(24)

is independent of x and Y_j . The differential equation that results is homogeneous, and can be integrated analytically for q = 3/2. This case will be dealt with below in some detail because the basic features of the solution carry over for all 1 < q < 2, for which solutions can only be obtained numerically.

4.1. The One-Volatile Planet

For the remainder of this section we will assume a onevolatile planet. The subscript j will be dropped, and Y_j/Y_a will be set to unity. Equation (23) is then

$$\frac{dY}{dx} = AY^{2-q}x^{q-2} - F_{\chi}y,$$
(25)

and the constant A_i simplifies to

$$A = \left\{ \xi_0 \left(\frac{3-q}{4-2q} \right) \right\}^{2-q} \left\{ F_\eta \frac{2-q}{q-1} - (1-F_\chi) y \right\}.$$
 (26)



FIG. 1. The maximum atmospheric mass Y that can be expelled by an impacting veneer of mass x for three values of q, the power law exponent describing the (differential) mass spectrum of impactors. Mars (dots) and even Venus (dashes) are quantitatively similar (both shown for q = 1.7). Rectangles indicate veneer masses (width corresponds to plausible q) needed to remove present atmospheres of Titan, Mars, and Venus.

4.1.1. Maximum impact erosion. Melosh and Vickery neglected the volatiles contributed by the impactor (y = 0) and assumed that half the impacts were energetic enough to effect erosion ($F_{\chi} = 0.5$). The assumptions $F_{\chi} = y = 0$ maximize the effect of impact erosion; therefore it is useful to first solve Eq. (25) with $F_{\chi} = 0$. This is equivalent to assuming that all large impacts are erosive; i.e., that $v > v_c$ for all impacts.

$$Y^{q-1} = Y_0^{q-1} - A\left(x_0^{q-1} - x^{q-1}\right)$$

= $Y_0^{q-1} - A\left\{\left(\frac{\delta M_0}{M}\right)^{q-1} - \left(\frac{\delta M(t)}{M}\right)^{q-1}\right\}.$ (27)

The initial state is at Y_0 and x_0 . Since $\delta M(t) \rightarrow 0$ as t approaches the present, retention of a finite atmosphere places an upper bound on the mass of veneer δM_0 that can be accreted. Accordingly, the minimum mass veneer x_0 needed to strip a planet of an atmosphere Y_0 is

$$x_0 = Y_0 A^{-1/(q-1)}.$$
 (28)

Figure 1 shows the biggest atmosphere Y_0 that can be completely removed by a veneer of mass x_0 . It has been prepared using Titan's parameters, but because $A(q, F_{\chi} = 0, y = 0)$ is comparable for the terrestrial planets, the corresponding lines for Mars and even Venus (the dashed line on Fig. 1 is for Venus with q = 1.7) are quantitatively similar. It should be recalled that x_0 refers to the mass of impactors incident on the planet; the mass actually retained is in this example very much less. It is interesting to consider how thin a veneer could strip Titan and Mars of their present atmospheres. These are indicated by the rectangles for q ranging from 1.5 to 1.83. For Titan ($Y = 6.6 \times 10^{-5}$) with 1.7 < q < 1.9, x_0 is about 0.002, while for Mars ($Y \approx 3.6 \times 10^{-8}$) with the same range of q, it is only of order 10^{-6} . The former corresponds to \sim 3 km of ice. The latter, which corresponds to a rocky veneer less than 2 m thick, highlights the extreme rarity of the Martian atmosphere. These estimates become sensitive to q for $q \le 1.6$; if for Mars we take MV's parameters $(F_x = 0.5 \text{ and } q = 1.47)$, the required rocky veneer rises to \sim 160 meters (mostly supplied by a single low velocity impact). We will discuss Mars more fully in a separate paper; for the present it suffices to note that the present Martian atmosphere is probably too thin to be comfortably explained as the end point of a history of impact erosion. That such thin veneers could separate planets from their atmospheres rather strongly suggests that the present distribution of planetary atmospheres has been greatly influenced by events at the very end of planetary accretion.

4.1.2. Impact erosion with competing source terms. In the more general case, with $F_{\chi} \neq 0$ and $y \neq 0$, Eq. (25) can be solved analytically only for q = 3/2.

$$\frac{dY}{dx} = A Y^{1/2} x^{-1/2} - F_{\chi} y.$$
 (29)

The constant A reduces to

$$A = (1 - F_{\chi}) \left(\frac{3H}{4R}\right)^{1/2} (1 - y).$$
 (30)

Equation (29) is solved by defining $u^2 \equiv Y/x$. The resulting equation for u is

$$\frac{2udu}{u^2 - Au + F_x y} = -\frac{dx}{x}.$$
(31)

The equation has three solutions, depending on whether the two roots of the quadratic equation obtained by setting the denominator equal to zero,

$$f(u) \equiv u^2 - Au + F_y y = 0, \qquad (32)$$

are real and separate, real and coincident, or complex. In a rather broad and inexact sense, the first category describes atmospheric histories that are fundamentally "erosive" and the third category histories that are "accu-



FIG. 2. Schematic illustration of atmospheric evolution in the erosive regime (a) and the accumulative regime (b). At any one time, x refers to the veneer mass still to be accreted. In the erosive regime (a), if the initial atmosphere is thinner than Y^+ , there results a thin atmosphere balancing impact erosion with impact delivery. This equilibrium atmosphere, Y^- , exists only during accretion, and declines to zero linearly with x. If the initial atmosphere is thicker than Y^+ , the veneer is not massive enough to expel all of the atmosphere. A thin atmosphere results only if the veneer mass is finely tuned to the initial mass of the atmosphere. In the accumulative regime (b), some atmosphere, corresponding to the evolution of an initially airless body, is described by Eq. (40).

mulative"; the second category marks the boundary between these.

If $A^2 > 4F_{\chi}y$, the two roots to Eq. (32) are real and separate. These are erosive histories, in that impact erosion (as represented by A^2) exceeds atmophile accretion $(4F_{\chi}y)$. The solution is expressed in terms of Q, defined by

$$Q \equiv \sqrt{A^2 - 4F_{\chi}y}.$$
 (33)

Although it may not be immediately apparent, the solution to Eq. (29) can be written

$$\begin{cases} \frac{Y^{1/2}(x) - \frac{1}{2}(A+Q)x^{1/2}}{Y_0^{1/2} - \frac{1}{2}(A+Q)x_0^{1/2}} \end{cases}^{A+Q} \\ = \left\{ \frac{Y^{1/2}(x) - \frac{1}{2}(A-Q)x^{1/2}}{Y_0^{1/2} - \frac{1}{2}(A-Q)x_0^{1/2}} \right\}^{A-Q}. \tag{34}$$

The initial state is denoted by $t = t_0$, $x_0 \equiv x(t_0)$, and $Y_0 \equiv Y(t_0)$. The roots are

$$\hat{Y}^{\pm} = \frac{1}{4} \left(A \pm Q \right)^2 x. \tag{35}$$

The two roots divide the (x, Y) plane into three regions. This is illustrated schematically by Fig. 2a. After some contemplation of Eq. (34) it becomes apparent that Y(x)can never cross a root. Thus there are three kinds of erosive histories, determined by the initial conditions and thereafter bounded by the roots: (i) If Y_0 is initially smaller than the smaller root \hat{Y}_0^- , then Y(x) will always be smaller than $\hat{Y}^-(x)$. These are the solutions in the lower third of Fig. 2a. These atmospheres vanish as x goes to zero, since $\hat{Y}^- \propto x$. Physically, these solutions correspond to the history of an atmosphere surrounding an initially airless planet in an intense atmospheric cratering regime. The mass of atmosphere asymptotically approaches the smaller root $\hat{Y}^-(x)$, which corresponds to the equilibrium atmosphere balancing loss and gain. But \hat{Y}^- is a vanishing equilibrium, since it declines to zero in proportion to the mass of veneer still to be accreted. Such ephemeral, accretion-dependent atmospheres may actually have some relevance in the solar system including, in particular, Mars and Callisto.

(ii) Solutions in the middle third of Fig. 2a begin with $\hat{Y}_0^- < Y_0 < \hat{Y}_0^+$. These solutions are bounded by the two roots, evolving away from \hat{Y}^+ and toward \hat{Y}^- as $x \to 0$. These atmospheres are also destined to disappear. The larger root \hat{Y}^+ describes the evolution of the thickest atmosphere that can be eroded to zero by a veneer of mass δM_0 .

(iii) Solutions in the top third of Fig. 2a are those for which $Y_0 > \hat{Y}_0^+$; i.e., these solutions correspond to initial atmospheres too thick to entirely erode away. In general the remnant atmosphere will then itself be thick, unless the initial conditions are very finely tuned. The final thickness of the remnant atmosphere is

$$Y_{\rm f} = \left\{ Y_{0}^{1/2} - \frac{1}{2} (A + Q) x_{0} \right\}^{1 + A/Q} \\ \times \left\{ Y_{0}^{1/2} - \frac{1}{2} (A - Q) x_{0} \right\}^{1 - A/Q}.$$
(36)

There does exist a range of initial conditions that can give rise to a thin remnant atmosphere, but the range is narrow. A more quantitative discussion of this matter is deferred to the Mars paper.

If $A^2 < 4F_X y$, the two roots to Eq. (32) are complex. Although we have labeled histories in this general class as accumulative, because the source 4 $F_X y$ exceeds erosion A^2 , they too can have a markedly erosive character. However, they differ fundamentally from the erosive histories in that an atmosphere is always left behind at the end of accretion. The parameter Q' is defined as

$$Q' \equiv \sqrt{4F_{\chi}y - A^2}.$$
 (37)

The solution to Eq. (31) is then

$$\log\left(\frac{Y - AY^{1/2}x^{1/2} + F_{\chi}yx}{Y_0 - AY_0^{1/2}x_0^{1/2} + F_{\chi}yx_0}\right) =$$

$$-\frac{2A}{Q'}\left\{\tan^{-1}\left(\frac{2Y^{1/2}-Ax^{1/2}}{Q'x^{1/2}}\right)-\tan^{-1}\left(\frac{2Y^{1/2}_{0}-Ax^{1/2}_{0}}{Q'x^{1/2}_{0}}\right)\right\}.$$
(38)

There is but one regime, as illustrated by Fig. 2b. Some atmosphere is retained whatever the initial conditions. The asymptotic solution as $x \rightarrow 0$ is

$$Y_{\rm f} = (Y_0 - AY_0^{1/2}x_0^{1/2} + F_{\chi}yx_0) \\ \times \exp\left[-\frac{2A}{Q'}\left\{\frac{\pi}{2} - \tan^{-1}\left(\frac{2Y_0^{1/2} - Ax_0^{1/2}}{Q'x_0^{1/2}}\right)\right\}\right].$$
(39)

The minimum atmosphere, corresponding to an initially airless planet, is

$$Y_{\rm f} = F_{\chi} y x_0 \exp\left[-\frac{2A}{Q'}\left\{\frac{\pi}{2} + \tan^{-1}\left(\frac{A}{Q'}\right)\right\}\right].$$
 (40)

This is a useful and tractable result. Equation (40) can also be applied to accretion of several volatiles provided that they maintain the same constant ratios in the atmosphere and the source impactors. This will be true if they are all strict atmophiles. If this condition is met, A and Q' in Eq. (40) are replaced by A_j and Q'_j ; A_j being obtained from Eq. (24) with q = 1.5, and Q'_j being obtained by using A_j rather than A in Eq. (37). We will use both versions of Eq. (40) later in our story of how Titan got its atmosphere (if one wishes to glance ahead, see Fig. 3).

In the special case where $A^2 = 4F_{\chi}y$, the two roots are real and coincident. In this case the solution to Eq. (31) is

$$\log\left(\frac{Y^{1/2} - \frac{1}{2}Ax^{1/2}}{Y_0^{1/2} - \frac{1}{2}Ax_0^{1/2}}\right) = \frac{Ax}{2Y - Ax} - \frac{Ax_0}{2Y_0 - Ax_0}.$$
 (41)

There are two regimes. If $2Y_0 < Ax_0$, then 2Y < Ax for all x, and the atmosphere disappears. These solutions correspond to atmospheres that are too thin initially to survive the bombardment. Thicker initial atmospheres survive; the remnant is

$$Y_{\rm f} = \left(Y_0^{1/2} - \frac{1}{2}Ax_0^{1/2}\right) \exp\left(-\frac{Ax_0^{1/2}}{2Y_0^{1/2} - Ax_0^{1/2}}\right).$$
 (42)

4.2 Why is there Air? The Division between Planets with and without Atmospheres

The above considerations imply that a planet or satellite can retain a substantial atmosphere if one of two conditions is met. The first occurs if the initial atmosphere is thick enough to survive impact erosion. This requires that

$$Y_0 > \dot{Y}_0^+ = \frac{1}{4} (A + Q)^2 x_0.$$
(43)

A special case of Eq. (43) was briefly discussed in the context of Fig. 1. The second condition occurs if atmospheric evolution is accumulative; i.e., if $A^2 < 4F_{\chi}y$. Because the first condition requires generation of a thick atmosphere in the first place, the second condition is more fundamental. The second condition can be placed either on F_{χ} , the fraction of slow impactors, or on y, the intrinsic atmophile content of the impactors. Since A is a function of y and F_{χ} (using Eq. (30) with $F_{\eta} = 1 - F_{\chi}$), for a given value of y the critical value F_{χ} is found by solving

$$2\sqrt{yF_{\chi}} - A(F_{\chi}, y) = 0.$$
 (44)

For a given value of y this is a quadratic equation for F_{χ} , or alternatively, for a given value of F_{χ} this is a quadratic equation for y. Equation (44) is not difficult to solve, but it is more useful to discuss the generalization to $q \neq 3/2$.

4.2.1. Extension to 1 < q < 2. The above argument regarding the impact generation of an atmosphere over an initially barren planet can be extended to other values of q. Consider the generalization of Eq. (31) obtained from Eq. (25) with $u^2 \equiv Y/x$:

$$\frac{2udu}{u^2 - Au^{4-2q} + F_{\chi}y} = -\frac{dx}{x}.$$
 (45)

Setting the denominator of the integrand on the left-hand side to zero yields

$$u^{2} - Au^{4-2q} + F_{y}y \neq f(u) = 0.$$
(46)

For $1 < q \le 2$, Eq. (46) has two real positive roots, one real positive root (i.e., the two roots are coincident), or no real positive roots. These cases are directly analogous to the different cases for q = 3/2.

When two positive roots exist, the larger corresponds to the repulsive equilibrium \hat{Y}^+ , and the smaller to the attractive equilibrium \hat{Y}^- . As with q = 3/2, the two roots decline linearly with x, so that unless the initial atmosphere Y_0 is thicker than \hat{Y}^+x_0 , the atmosphere vanishes as $x \to 0$. The larger root gives the maximum atmosphere that can be stripped by a veneer of mass x. Also as with q = 3/2, when there are no positive real roots to f(u) = 0, at least some atmosphere must accumulate.

Again by analogy to the case with q = 3/2, the critical solution—the solution dividing airless planets from those accumulating atmospheres—is that solution for which f(u) = 0 has one real positive root. Since in this critical case the one real positive root of f(u) = 0 is a double root, the first derivative of f(u) is also equal to zero. Put more graphically, the critical solution is the one where the mini-

mum value of f(u) is zero. Thus the root is easily found by solving df/du = 0 subject to the condition that f(u) = 0. Eliminating *u* gives the desired relationship between *q*, F_y , and *y*,

$$(2-q)A = \left(\frac{2-q}{q-1}F_{\chi}y\right)^{q-1}.$$
 (47)

Equation (47) defines the boundary between the accumulative and erosive regimes. The constant A, as defined by Eq. (26), is a function of several parameters, including F_{χ} and y. Thus Eq. (47) is an implicit equation for the critical value of F_{χ} given y and q, or alternatively, an implicit equation for the critical value of y given F_{χ} and q. Solutions to Eq. (47) must be found numerically. Equation (47) can be used for a multivolatile planet if A is replaced by A_i , as defined by Eq. (24). The argument leading to Eq. (47) is rigorously valid in the limit that the dominant reservoir for all major atmospheric constituents is the atmosphere itself. For a fixed value of q, the key factors that determine whether an initially barren satellite or planet will accumulate an atmosphere are the size of the planet, the impact velocity distribution F_{y} , and the innate volatile content of the impactors y. The size of the planet enters directly as R and indirectly as the atmospheric scale height H, but more importantly, it enters indirectly in F_{y} through $v_{\rm esc}$.

5. TITAN, GANYMEDE, AND CALLISTO

There are several ways we might attempt to construct divergent evolutionary histories for the three satellites. A traditional approach is to postulate different volatile inventories in circum-Jovian and circum-Saturnian "satellitesimals" [e.g., Lunine et al. 1989]. It could reasonably be argued, for instance, that the stuff Titan was made from was much colder than the stuff Callisto and Ganymede were made from, and thus was qualitatively more volatile-rich. In particular, this hypothesis would seem to require that Titan contain both ammonia and methane as important bulk constituents. Lunine et al. (1989) give two examples of such scenarios. One is that Titan, but not Ganymede and Callisto, accreted CH₄ and NH₃. The other allows Ganymede and Callisto to accrete NH₃, but not CH₄. The latter can be accomodated if CH₄ were the critical ingredient in a coaccreting greenhouse atmosphere. Then the presence of CH₄ on Titan might have allowed NH₃ to become an atmospheric species on Titan alone. Once in the atmosphere NH₃ could have been converted to N₂ by photochemistry or shock chemistry. Neither is an unattractive concept, but both would seem to predict more than a surficial compositional difference between Titan on the one hand and Ganymede and Callisto on the other.

A second approach is to postulate preexisting thick atmospheres, and then ask whether a late veneer of high velocity impactors can selectively remove them (e.g., Lunine et al. 1989). This begs the question of the origin of the atmospheres in the first place, although accretion of a primary atmosphere from a swarm of slow-moving titanesimals (or their counterparts in the Jovian system) seems plausible enough (Lunine et al. 1989). But there is another matter. If this earliest atmosphere is to have any relevance to the modern solar system, it must survive later bombardment by high velocity intruders. We showed earlier that if both $F_{\chi} \ll 1$ and $y \ll (2 - q)/(q - 1)$, a late veneer equivalent to 3 km of ice could suffice to strip Titan of its present atmosphere. This was the subject of Fig. 1. Three kilometers of ice is not a lot of material. It would be surprising if the late veneer were so thin. Moreover, to strip Callisto and not strip Titan would require a veneer of just the right thickness-neither much thicker, ere both be stripped, nor much thinner, ere neither be stripped. Such fine-tunning makes us uncomfortable, as it did (in somewhat different circumstances) Lunine et al. before us. If fine-tuning is to be avoided, the restrictions on either F_{χ} or y must be relaxed. If just the restriction on F_{χ} is relaxed, then fine-tuning can be avoided if F_{y} is small for Callisto but close to one for Titan. If the restriction on y is relaxed, the veneer necessarily contributes heavily to the atmosphere, and comes to dominate its composition. This latter case is ultimately indistinguishable from the case where an atmosphere accretes around an initially barren object.

This brings us to the path we will follow here. We will begin with the artificial assumption that the satellites were initially barren, and that Titan's atmosphere is a by-product of a late-accreting "cometary" veneer. Because for any single source q and y would be the same for all three satellites, F_{χ} is the only parameter that distinguishes among them. We will consider three potential sources for the comets: Uranus-Neptune planetesimals, the (hypothetical) Kuiper belt, and the Oort cloud. The location of the source affects the impact velocity. If two of these sources were important in differing degrees for the different satellites, some variance in q and y might result.

An attractive feature of this late veneer hypothesis is that it automatically accounts for the general compositional similarity shared by the atmospheres of Titan, Triton, and Pluto. The explanation could be straightforward—they were coated with the same stuff—but a subtler explanation is more appealing; Triton and Pluto may themselves represent the composition of the veneering material. Triton, which is apparently a captured satellite of Neptune, and Pluto, which is probably not an escaped satellite of Neptune, are sufficiently alike (in particular, they have similar densities, which are high relative to outer Solar System regular satellites) as to suggest that they are surviving representatives of the once plentiful planetesimals that formed Uranus and Neptune. It has long been suspected that comets are also surviving U–N planetesimals. Such outer Solar System objects are natural candidates for a late veneer.

Because there are four poorly constrained parameters—three parameters $(q, y, \text{ and } F_{\chi})$ determine whether an atmosphere accumulates, and a fourth, the veneer mass x_0 , affects the quantity of atmosphere that accumulates—it is difficult to formulate a telling test of our hypothesis. We cannot expect to reach a clear, unambiguous decision.

We will first discuss the atmophile content of the impactors, and then ask whether Titan's atmosphere can be supplied by a veneer of reasonable thickness. We will then map out the surface in parameter space between objects that accumulate atmospheres and those that do not. For presentation, we will at first treat y and q as independent and F_{χ} as dependent. Since F_{χ} is the only parameter that effectively distinguishes between Titan, Ganymede, and Callisto, we will present these results for Titan only. We will then attempt to calculate F_{χ} for the three satellites. Finally, we will consider where Ganymede, Callisto, and Titan place with respect to this boundary.

5.1. The Atmophile Content of the Impactors

In the present context the relevant atmophiles are probably N_2 , CH_4 , and CO. Water cannot be regarded as an atmophile or even a volatile at these distances, and in all likelihood the same prohibition applies against NH₃, HCN, and CO_2 . However, what is important in the present context is not the form taken by the atmophile elements while in the comet, but instead the form they take after impact, since it is in this form that they enter the atmosphere. In a low speed impact, such as those of the titanesimals, it is imaginable that a relatively fragile molecule like NH₃ might survive impact. But in the high speed impact of a comet or U-N planetesimal very high temperatures and pressures are unavoidable (e.g., Chyba et al. 1990), and most molecules are torn apart. A complete chemical reorganization in accord with high temperature thermochemical equilibrium takes place. As the shocked gases cool chemical reactions take place more slowly, and those with significant activation energies completely stop, leaving the characteristic chemical composition of hot gases "frozen" in (Zel'dovich and Raizer 1967, pp. 564-571).

For a more-or-less solar composition comet, the most important C- and N-bearing products are likely to be CO and N₂, with small additional amounts of CO₂, C₂H₂, CH₄, and HCN. Elemental carbon might also form. For nitrogen this leads to a simple resolution: whatever form it took in the comet, most of the comet's nitrogen enters the atmosphere as N_2 . Carbon is more problematic; suffice it to say that CO atmospheres no longer exist, perhaps having been consumed by reaction with water to form CO_2 , CH_4 , and H_2 , the latter readily escaping to space.

Compositional data for Comet Halley (Jessberger et al. 1989) can be combined with solar elemental abundances (Anders and Grevasse 1989) to provide useful constraints on plausible values of y for planetesimals formed in the outer Solar System. The major elements that constitute these objects are Si, Mg, Fe, S, O, C, N, and H. We assume that all the sulfur combines with Fe to form troilite (FeS), and that the rest of the rock-forming elements (Si, Mg, and the balance of the iron) combine stoichiometrically with oxygen. We assume that water ice contains about 50% of the cosmic complement of O and that "CHONs" (chemically complex particles composed chiefly of the elements C, H, O, and N, in the approximate ratios 1:1:0.5:0.12) contain about 55% of the cosmic complement of C (see Pollack et al. 1991 for a full account).

The balance of the cosmic complement of C, N, and O would have been present in highly volatile species, chiefly CO, which do not appear to have fully condensed where Halley formed. In situ and remote sensing data from the Giotto and VEGA spacecraft observations of Halley provide abundant estimates for the major volatile molecules in the coma. These are given in terms of gas production rates relative to H₂O (Jessberger et al. 1989). Carbon monoxide was the most abundant C-bearing molecule, produced at $\sim 17\%$ of H₂O. About half of it derived from the decomposition of some more complicated parent, but the rest appears to be CO native to the comet. With the above assumptions, our standard comet is (by mass) roughly 34% water ice, 31% rock, 23% CHON, and 9% CO and parent, with the balance consisting of other volatiles present in small amounts. Ammonia, N₂, and CH₄ are of particular interest. Ammonia was an important N-bearing volatile, and is thought to be a parent molecule. If so, it represents a mass fraction of about $1-3 \times 10^{-3}$. Molecular nitrogen was not detected but is expected in some theories at the percent level (Engel et al. 1990). Finally, CH₄ was detected and is thought to have been derived from a more complicated precursor with a fractional abundance of order 7 \times 10⁻³.

As CHON particles are the major reservoirs of C and N in comets, additional C- and especially N-bearing volatiles are likely to be produced by shock-heating of planetesimals when they strike a satellite. If all the C and N in CHONs were liberated on impact as CO and N_2 , this would correspond to mass fractions of 0.16 and 0.016, respectively. In summary, for nitrogen y probably lies between about 0.001 and 0.03, while for carbon y probably lies between 0.04 and 0.25. The lower values refer to

volatile parent molecules only (i.e., NH_3 and CO). The upper limits include the easily converted parent molecules and complete conversion of CHON particles to atmophiles, as well.

5.2. How Thick a Veneer is Needed to Give Titan Its Atmosphere?

The first question to ask is whether Titan's atmosphere can be supplied by a veneer of reasonable thickness. This question concerns a fourth free parameter, x_0 , that does not figure in the *qualitative* matter of whether an atmosphere accumulates but does figure in the quantitative matter of how much atmosphere accumulates. The predictions of Eq. (40) for an initially airless Titan are shown in Figs. 3a and 3b. Figure 3a was prepared presuming the Titanian $Y_f = 6.6 \times 10^{-5}$ (its present atmosphere). Equation (40) was then solved for the veneer mass x_0 as a function of the parameter F_{χ} . (Recall that x_0 refers to the total mass of incident impactors; the mass actually accreted is of order $F_{y}x_{0}$). The different curves correspond to different volatile contents y. The same figure applies with only minor adjustments for the accretion of Titan-like atmospheres by Callisto or Ganymede. Equation (40) presumes q = 1.5. These are the solid curves in Figs. 3a and 3b. Thicker veneers are needed for q = 1.7. For comparison, analogous curves with q = 1.7 for y =0.01 and y = 0.03 are shown as dotted lines on Fig. 3a. These were obtained by numerically integrating Eq. (25).

It is apparent that it would have been difficult for Titan to accrete its atmosphere from a thin veneer of material if $F_{\chi} \ll 1$. This highlights a fact that is worth stressing here: Titan's atmosphere is quite thick. Titan is much more like Venus ($Y = 9 \times 10^{-5}$) than Earth ($Y = 8.5 \times 10^{-7}$). If one is simply comparing nitrogen inventories, Titan is vastly richer than either. It is not easy to generate such a thick atmosphere from a thin veneer, but it can be done.

Figure 3a assumes that Titan's atmosphere was always predominantly nitrogen. This may be too pessimistic. It is possible that during accretion other more abundant atmophiles resided in the atmosphere. This would make the coaccreting atmosphere thicker than if it were N₂ alone. Since a thicker atmosphere is less subject to impact erosion, this would make it easier to retain nitrogen than if N_2 were the only important atmophile. Because it is potentially abundant in cometary material and a reasonable product of shock-heating (Prinn and Fegley 1989), carbon monoxide may be the best candidate for a transient atmosphere. Equation (40) can be applied to accretion of two volatiles provided that their ratio is constant and A and Q' are replaced by A_j and Q'_j . Figure 3b has been prepared assuming a (high) constant CO/N₂ ratio of 30 in both the atmosphere and the accreting material. This ratio



FIG. 3. The veneer mass needed to produce Titan's atmosphere. The curves are labeled by the impactor atmophile content y and are shown as a function of the parameter F_{χ} , the fraction of impacts with impact velocity below v_c . (a) For the one-atmophile planet. In this case the atmophile is N₂. Solid curves denote q = 1.5; dotted curves q = 1.7. (b) For a two-atmophile planet. It is assumed that Titan accreted CO and N₂ in the ratio 30: 1. To facilitate comparison with (a), in (b) y refers to the nitrogen content of the impactors. Note that x refers to the incident veneer mass; the mass actually retained is of order xF_{χ} .

is comparable to the surficial CO_2/N_2 ratios on the terrestrial planets. The key difference between the results shown in Figs. 3a and 3b is that, if impact erosion is important (F_{χ} small), it is easier to accrete a Titan's worth of nitrogen under a carbon monoxide shield.

5.3 Under What Conditions Does Titan Get an Atmosphere?

Figures 4 and 5 show minimum values of F_{χ} for which Titan develops an atmosphere. In Fig. 4, the critical value of F_y is shown as a function of q for four values of y, the atmophile content of the impactors. In Fig. 5 the roles of y and q are reversed. In preparing Figs. 4 and 5, Eq. (47)was solved for Titan, but because the three satellites have similar escape velocities, the corresponding curves for Callisto and Ganymede are practically indistinguishable from Titan's. In both these figures y refers not just to nitrogen but to the total atmophile content of the impactors. It is apparent from the figure that, unless y were at the high end of this range (or q extreme), an atmosphere would not be expected to accumulate around any of these icy satellites unless F_{χ} were pretty big; i.e., unless the average impact velocity were comparable to or lower than v_{c} . As will be shown below, this condition is difficult to meet for Callisto and Ganymede, but not unreasonable for Titan.

The dependence on q seen in Fig. 4 of F_{χ} is interesting. These calculations suggest that it is easier to capture an atmosphere from a population of impactors with an ex-



FIG. 4. The boundary between Titans that get atmospheres and Callistos that do not. This boundary is a surface in the three parameter F_{χ} , y, q space. Here F_{χ} , which is the only one of the three to differ significantly between Callisto and Titan, is treated as the dependent variable. q is the independent variable; the different curves are labeled by y. In this and subsequent figures y refers to the total atmosphere content, including C-bearing volatiles. Atmospheres accumulate for F_{χ} values above the appropriate curve.



FIG. 5. Like Fig. 4, but the roles of q and y have been exchanged. Atmospheres accumulate above the curves. An atmosphere always accumulates if y > (2 - q)/(q - 1). Figures 4 and 5 imply that for reasonable values of q and y, Titan would accumulate an atmosphere if $F_{\chi} \approx 0.5$. Unless y were large or q extreme, atmospheres do not accumulate for $F_{\chi} \ll 1$.

treme value of q, be it high or low, than from one with an intermediate value of q. The reasons for this differ for q high and low, although both are traceable to the same cause: the chief agents of atmospheric erosion are the smallest impactors that can effectively expel atmosphere, as these are the most numerous. These are the impactors with mass of order m_c .

When q is smaller than 2, most of the mass is in the larger objects. The lower the value of q, the more strongly the total mass of all the impactors is concentrated in the few largest objects. If these large bodies are as volatilerich as the other impactors, and if they are as efficiently degassed on impact (which are the assumptions made in this paper), they supply practically all the atmophiles. But very large impactors are relatively inefficient atmospheric craterers, because the damage done by any individual impactor is limited to expelling the atmosphere above the tangent plane. With a low value of q there are fewer impactors with masses of order m_c for a given mass of veneer accreted, and therefore less impact erosion for a given mass of atmosphere accreted. It should also be noted that for very low values of q chance plays a bigger role, because the final state could depend on the timing of the last large, slow-moving object to hit.

Objects which are too small to cause atmospheric cratering are assumed to contribute their atmophiles to the atmosphere. For higher values of $q \ (q \rightarrow 2)$, the mass accreted in small impactors becomes a nonnegligible fraction of the total. Note that if q is large and the impactors especially atmophile-rich, thick atmospheres are predicted even for $F_{\chi} \ll 1$, and therefore, a thick atmosphere would accumulate whatever the satellite's mass. This occurs because all impactors smaller than m_c are regarded as captured, regardless of impact velocity.

Figure 5 shows some of the same information as Fig. 4, but in this case with y the independent variable and q the label. Note that if y exceeds (2 - q)/(q - 1) an atmosphere accumulates for any F_{χ} . Mathematically, this particular value of y is important because A changes sign. Physically, this is the same phenomenon (volatiles supplied chiefly by small impactors) discussed in the previous paragraph, seen from a different perspective. This is potentially an interesting limit in contexts where water can be regarded as an atmophile, since y for comets would then be of order 0.5.

5.4. Impact Velocities on Ganymede, Callisto, and Titan

To get F_{χ} , the cumulative impact velocity distribution must be compared to the velocity v_c at which impact erosion begins. The velocity at which comets strike a satellite depends on the heliocentric orbit of the comet, the satellite's depth in the planet's potential well, and the surface escape velocity from the satellite. We use an Öpiklike approximation, which breaks the motion into three successive two-body problems (Öpik 1976, Shoemaker and Wolfe 1982). In this approximation the impactor moves only under the influence of solar gravity until it crosses the planet's Hill sphere (which has a radius of roughly 0.5–0.6 AU for Jupiter and Saturn), then only under the planet's gravity, and finally under the satellite's gravity. This should be a good approximation for orbits with sizeable heliocentric eccentricity or inclination.

A crude approximation to the mean-square impact velocity useful for order of magnitude comparisons is (Lissauer *et al.* 1988, Eq. (21); see also Eq. (52) below)

$$v_{\rm rms}^2 \approx v_{\infty}^2 + 3v_{\rm sat}^2 + v_{\rm esc}^2,$$
 (48)

where v_{sat} is the orbital velocity of the satellite about the planet; v_{esc} is the surface escape speed from the satellite; and v_x is the relative velocity of the planet and the comet. The last satisfies

$$v_{\infty}^2 = U^2 v_p^2 = v_p^2 (3 - 1/a - 2\cos i \sqrt{p(2 - p/a)}),$$
 (49)

where U^2 is a dimensionless parameter between 0 and 3 defined by Eq. (49); v_p is the planet's orbital velocity about the Sun; *a* and *p* are respectively the semimajor axis and

	ν _p	v _{sat}	U _{esc}
Ganymede	13.06	10.88	2.74
Callisto	13.06	8.19	2.45
Titan	9.64	5.57	2.64

perihelion distance of the comet in units of the planet's semimajor axis; and *i* is the inclination of the comet's orbit from the planet's orbit plane. Circular orbits are assumed for both satellite and planet. Equation (48) is an approximation because it neglects the relationship between impact probability and impact velocity. In particular, orbits with low impact velocities tend to have high impact probabilities; hence Eq. (48) overestimates $v_{\rm rms}$. The magnitude of the overestimate is typically a few kilometers per second (see Shoemaker and Wolfe 1982). In Table I we list relevant parameters for Ganymede, Callisto, and Titan. Velocities are given in kilometers per second. In Table II we give the extreme values of v_{∞} and $v_{\rm rms}$ for Uranus-Neptune planetesimals, Kuiper belt comets, and Oort cloud comets. These velocities, which were estimated using Eqs. (48) and (49), are shown only to illustrate the range of impact velocities expected. They are calculated for U-N and Kuiper belt planetesimals for $0 \le i \le 10^{\circ}$ and a = 25 AU and a = 50 AU, respectively; the minimum impact velocities occur with p = 1 (perihelion at the planet), and the maximum with p = 0. For Oort cloud comets we used a = 20,000 AU; the maximum impact velocities occur with $i = 180^{\circ}$ (retrograde orbits) and p =1, and the minimum occur with $i = 0^{\circ}$ and p = 1.

What we really need to determine F_{χ} is the distribution of impact velocities. To obtain this we must define the orbital distributions of the potential impactors, and then determine the impact velocity and impact probability associated with each orbit. Accordingly, we have written a computer program to implement the following algorithm: (1) First we select the heliocentric distributions of a, p, and i for the impactors. We assume the number density $n(a, p, i) da dp di \propto \sin i da dp di$, so that a is uniformly distributed between a_{\min} and a_{\max} , p is uniformly distributed between p_{\min} and p_{\max} , and $\cos i$ is uniformly distributed for inclinations below i_{\max} . This inclination distribution is uniform in solid angle between the minimum and maximum values of $\cos i$. For each orbit, we calculate v_{∞} using Eq. (49).

(2) We then consider the comet's motion near the planet. The value of v_{∞} is the comet's only memory of its previous heliocentric existence. We must determine which impact parameters b allow a comet to impact a satellite at a given orbital radius. For simplicity, we express the encounter using units in which the satellite's semimajor axis is 1. By conservation of angular momentum, $bv_x = rv_1$, where v_1 is the tangential component of the comet's orbital velocity about the planet at distance r. Collisons are possible for any value of b from zero (a radial orbit) to $b_{\text{max}} = rv/v_{\infty}$, where $v^2 = v_{\infty}^2 + 2v_{\text{sat}}^2$ is the comet's total orbital velocity squared; the latter case corresponds to periapse at the satellite's orbital radius. Thus $b_{\text{max}}^2 = 1 + 2v_{\text{sat}}^2/v_{\infty}^2$. Since the gravity of the planet has little effect on the comet before it enters the Hill sphere (for large v_{x}), we assume that the distribution of impact parameters is uniform in area: $n(b)db \propto bdb$, or $n(b^2) \propto d(b^2)$ between limits of 0 and b_{max}^2 .

The relative velocity of the comet and satellite (ignoring the gravity of the satellite for now) is

$$v_{\rm rel} = |\mathbf{v}_{\rm comet} - \mathbf{v}_{\rm sat}|,\tag{50}$$

where the velocity vectors of comet and satellite are $\mathbf{v}_{comet} = (v_r, v_t \cos i', v_t \sin i')$ and $\mathbf{v}_{sat} = (0, v_{sat}, 0)$, where $v_r = \sqrt{v^2 - v_t^2}$ is the comet's radial velocity, and *i'* is its inclination with respect to the orbital plane of the satellite. Simplifying Eq. (50), we obtain

$$v_{\rm rel}^2 = v_{\infty}^2 + 3v_{\rm sat}^2 - 2v_t v_{\rm sat} \cos i'.$$
 (51)

impact velocities on Satemets						
	Uranus-	-Neptune	Kuip	er belt	С	Port
	$ \begin{array}{r} 25 \text{ AU} \\ (0 < i < 10^{\circ}) \end{array} $		$50 \text{ AU} \\ (0 < i < 10^{\circ})$		$\begin{array}{c} 20,000 \text{ AU} \\ (0 < i < 180^{\circ}) \end{array}$	
Semimajor axis inclinations		v _{rms}		v _{rms}	Ux	U _{rms}
Ganymede	4.4-21.8	19.5-28.9	4.9-22.2	19.7-29.2	5.4-31.5	19.8-36.8
Callisto	4.4-21.8	15.1-26.1	4.9-22.2	15.2-26.5	5.4-31.5	15.4-34.6
Titan	2.6-15.6	10.3-18.5	3.3-16.2	10.5-19.0	4.0-23.3	10.8-25.4

TABLE II Impact Velocities on Satellites

The last assumption required is the distribution of i'. We make the crude assumption that the distribution is isotropic, i.e., that $n(i') di' \alpha d \cos i'$. Even if the heliocentric inclinations, i, of the impactors are low, the distribution of i' will extend to higher values because of the gravitational deflection of the comets by the planet and the planet's non-zero obliquity. The assumption of isotropy roughly accounts for aberration effects which cause impact velocities on prograde satellites to be somewhat lower at orbital longitudes on the sunward side of the planet, and higher at longitudes on the anti-sunward side (Cuzzi and Durisen 1990). Our main results are not highly sensitive to the assumption of isotropy.

(3) Finally, we treat the motion of the comet very near the satellite. In the two-body approximation, conservation of energy gives the final impact velocity as

$$v_{\text{impact}}^{2} = v_{\text{rel}}^{2} + v_{\text{esc}}^{2} = v_{\infty}^{2} + 3v_{\text{sat}}^{2} - 2v_{t}v_{\text{sat}}\cos i' + v_{\text{esc}}^{2}$$

$$\equiv v_{\text{rms}}^{2} - 2v_{t}v_{\text{sat}}\cos i'.$$
(52)

We then compute the distribution of impact velocities on a satellite numerically, instructing the computer to loop over the five varibles a, p, i, b, and i'. We weight the impact probabilities according to the formulation given by Shoemaker and Wolfe (1982). The collisional probability P_c is proportional to the factors

$$P_{\rm c} \propto \left(1 + \frac{2v_{\rm sat}^2}{v_{\rm x}^2}\right) \left(1 + \frac{v_{\rm esc}^2}{v_{\rm x}^2 + 2v_{\rm sat}^2}\right) \frac{U}{U_{\rm x} \sin i}, \quad (53)$$

where U and i are defined above, and U_x is the radial component of the dimensionless encounter velocity, $U_x^2 = 2 - 1/a - p(2 - p/a)$. The first factor represents gravitational focusing by the planet, the second factor is gravitational focusing by the satellite (unimportant), and the third the relative overlap of the object's orbit with the (gravitationally enhanced) torus swept out by the planet.

Using this formalism, we have computed impact velocity distributions for U-N planetesimals, Kuiper belt comets, and Oort cloud comets hitting Ganymede, Callisto, and Titan. We have assumed that perihelia are uniformly distributed between 0.10 and 5.10 AU (Jupiter) and 5.30 and 9.44 AU (Saturn); other parameters are listed in Table II. The perihelia for Saturn-crossers were chosen on the assumption that the perihelia must lie outside of Jupiter, because bodies on Jupiter-crossing orbits evolve rapidly into short period or long period cometary orbits. Median impact velocities for objects striking Titan, Callisto, and Ganymede are given in Table III. It is immediately apparent that there is little difference between Kuiper belt comets and U-N planetesimals. Clearly once the semimajor axis is well outside the planetary orbit, it makes very little difference to the impact velocities. By contrast, Oort

TABLE III Median Impact Velocities on Satellites

	Uranus-Neptune	Kuiper belt	Oort 27.5	
Ganymede	20.6	20.7		
Callisto	16.3	16.6	26.0	
Titan	11.1	11.3	19.1	

cloud comets generally have much higher impact velocities. This is because of their generally higher inclinations, which we have assumed isotropic.

Figure 6a shows the cumulative impact velocity distribution of U–N planetesimals striking the three satellites. The corresponding plot for Kuiper belt comets is essentially indistinguishable, and so will not be presented here. It would appear that impact erosion by either class of object can explain the differences between Titan and Callisto if $10 \le v_c \le 14$ km/sec. Impact velocities by Oort cloud comets, Fig. 6b, are higher, yet still show a significant low impact velocity tail.

5.5. v_c

We will idealize the impact of comets on icy satellites as the impact of water ice projectiles on water ice targets. Water is sufficiently abundant that it should dominate impact-generated vapor clouds at the relatively low impact velocities that are of interest here. As discussed earlier, Melosh and Vickery suggested that the minimum impact velocity for atmospheric erosion for identical impactor and target materials is

$$v_c = 2\sqrt{v_{\rm esc}^2 + 2L_{\rm vap}},\tag{1}$$

where $L_{\rm vap} = 3 \times 10^{10}$ erg/g is the latent heat of vaporization for water. Titan, Ganymede, and Callisto all have escape velocities of ~2.6 km/sec. For an icy body impacting an icy surface, Eq. (1) predicts that the threshold velocity for escape would be just ~7 km/sec.

However, Eq. (1) cannot be the whole story. Equation (1) assumes (i) normal impact; (ii) that the internal energy deposited by the shock is divided between the energy needed to vaporize the shocked material and sensible heat, and that all of the latter remains in place as thermal energy of the vapor; (iii) that there is no mixing of the hottest vapors with cooler vapors produced from more distant parts of the shocked target; and (iv) that the latent heat of condensation is unavailable to the expanding vapor plume (i.e., the vapor does not condense). The first three assumptions overestimate the thermal energy of the vapor, since (i) peak shock pressures are lower for oblique impacts (in an oblique impact it is the normal component

FIG. 6. Impact velocities of stray Uranus-Neptune planetesimals (a) and Oort cloud comets on Ganymede, Callisto, and Titan. Impact velocities of Kuiper belt comets are very similar to U-N planetesimals.

of the impact velocity that matters most; since the most probable impact angle is 45°, Eq. (1) may on average underestimate v_c by a factor $\sqrt{2}$); (ii) a fair fraction of the internal energy deposited by the shock is reversibly stored in the compressed material, to be returned as useful work (in this case, excavation) upon decompression (Zel'dovich and Raizer 1967, pp. 762–770); and (iii) there will be mixing. These effects are partially offset by the fourth assumption, since in practice it must be expected that some of the steam produced by the impact does condense relatively quickly.

A wholly reliable measure of v_c would require an accurate, comprehensive, numerical simulation of the cratering process with good equations of state. If we are willing to sacrifice some reliability, a usable rough estimate for the thermal energy of an impact-generated vapor cloud can be developed from arguments given elsewhere by one of us (Zahnle 1990). Following a simple argument crater-scaling given by Davies (1985), the for Schmidt-Housen (1987) and Schmidt-Holsapple (1982) relations for scaling crater diameter with impact parameters (in the large crater limit in which gravity dominates) can be cast in terms of the fraction h of internal energy deposited by the shock which remains behind as heat (0 < h < 1). The essence of the argument is the assumption that all the kinetic energy and a fraction 1 - h of the internal energy are available for further propagation of the shock, the kinetic energy as ram pressure (because the shock decelerates, the previously shocked material catches up) and 1 - h of the internal energy from the elastic part of decompression and to a lesser extent from thermal pressure. Note that we are interested not in the thermal energy immediately behind the shocked but instead in the thermal energy remaining after the shocked material has fully contributed to the further propagation of the shock. At this point the shocked material is no longer descending rapidly into the planet, but rather would be expected to have a center of mass velocity comparable to that of the excavation flow, i.e., probably no more than about one or two kilometers per second. In terms of h, the modified form of Eq. (1) is

$$v_c = \frac{2}{\sqrt{h}}\sqrt{v_{\rm esc}^2 + 2L_{\rm vap}}.$$
 (54)

The parameter h is related to Schmidt and Housen's parameter μ by $h = 2 - 3\mu$. An upper bound on v_c might be set by multiplying the above by $\sqrt{2}$ to account for obliquity. On the other hand, if the vapor condenses the latent heat of condensation can also contribute to expansion; accordingly, a lower bound on v_c is

$$v_c^* = \frac{2}{\sqrt{h}} v_{\rm esc}.$$
 (55)

This latter expression is clearly too low if $v_{esc}^2 \ge L_{vap}$, since it makes no allowance for the energy needed to vaporize water in the first place. An estimate for the minimum impact energy needed to completely vaporize the impactor is (Zahnle 1990)

$$v_{\rm vap} \approx \frac{4}{\sqrt{h}} \sqrt{L_{\rm vap}}.$$
 (56)





FIG. 7. Critical impact velocities v_c and v_c^* for escape and the critical impact velocity v_{vap} for complete vaporization are shown as functions of h, the fraction of internal energy initially deposited by the shock that remains as thermal energy after decompression. The higher velocity, v_c , assumes that latent heat of condensation does not contribute to the expansion velocity of the impact-generated vapor cloud, while v_c^* assumes that it does. The Schmidt-Housen (1987) crater-energy scaling relation is equivalent to h = 4/5 for porous materials and 1/3 for nonporous materials. Melosh and Vickery (1989) use v_c with h = 1. The specific calculations shown are for Titan, but are essentially the same for Callisto or Ganymede.

This expression, which through h implicitly allows for crater formation, is derived on the assumption that the threshold for vaporization is defined by the decompression adiabat of the shocked material passing through the critical point (Zel'dovich and Raizer 1967, pp. 762–770). Hotter release adiabats correspond to final states that are predominantly vapor; cooler adiabats correspond to final states that are mainly condensed phases. Zel'dovich and Raizer (1967) note that, in general, the thermal energy of vapor at the critical temperature is roughly equal to the latent heat of condensation at low temperature; i.e., complete shock vaporization requires that the shock deposit roughly twice the latent heat of vaporization. A more accurate estimate is $v_{vap} = 7.9$ km/sec at 263 K (McKinnon, personal communication); according to Eq. (56) this would correspond to h = 2/3 if taken at face value.

In Fig. 7 we show v_c , v_c^* , and v_{vap} for water ice impacts on an icy Titan as functions of the parameter *h*. These have not been corrected for obliquity, which would raise all three curves. The corresponding curves (not shown) are very similar for Callisto and Ganymede. Notably, values of v_{vap} given by Eq. (56) and v_c given by Eq. (54) are nearly the same, while v_c^* given by Eq. (55) is much lower. Because the latter is inconsistent with our presumption that the impactor is completely vaporized, we prefer the higher values of v_c .

What h should be in a strong shock is unclear. The Schmidt-Housen relation (for impact on a nonporous target; specifically, water-saturated sand) is equivalent to h = 1/3, and its predecessor, the Schmidt-Holsapple relation (porous target; dry sand), is equivalent to h = 4/5. A third choice can be derived from the observation that postshock particle velocities follow a power law in distance from the point of impact, $u \propto r^{-1.87\pm0.05}$ (Melosh 1989, p. 66). This is equivalent to $h = 0.40 \pm 0.05$ (Zahnle 1990). For the relatively large impacts of most interest to us, which excavate well below the porous regolith, porosity of the target material is likely to be relatively low. On the other hand, the porosity of the impactor may be rather high. Use of h = 1/3 would raise the threshold impact velocity for atmospheric cratering to ~ 12 km/sec. a value that should probably be regarded as an upper limit. A useful lower limit is obtained for h = 4/5, which gives $v_c = 9.0$ km/sec. Maximal equivocation is ensured by taking h = 1/2.

Figure 8a shows $F_{\chi}(h)$ obtained using Eq. (54) for $v_c(h)$ for Ganymede, Callisto, and Titan when the impactors are U–N planetesimals (the impact velocity distributions shown in Fig. 6a). Figure 8b is the analogous figure using v_c^* given by Eq. (55). In either case, results for Kuiper belt comets are similar. Figure 8c shows F_{χ} for Eq. (54) for v_c for Oort cloud comets. Some representative values are listed in Table IV for U–N planetesimals and Oort cloud comets.

Given values of F_{χ} appropriate to Callisto and Titan, Eq. (47) is solved for y. Figure 9 shows the minimum atmophile content y for which an atmosphere can develop around Titan and Callisto as a function of q. These are calculated for U–N planetesimals and v_c using the cases listed in Table IV; the analogous curves for Kuiper belt comets are essentially indistinguishable. The corresponding curves for Ganymede are not shown; its chances of getting an atmosphere are much worse than Callisto's. The best case for Callisto nearly coincides with the worst case for Titan. It is apparent that Callisto has no real chance of accumulating an atmosphere unless the highest estimate for v_c is adopted (low h) and q is either very high or very low. Titan, by contrast, would develop an atmosphere for any q if our higher estimates for v_c pertained; and would stand a fair chance of developing one for the lowest given a low value of q.

The expansive error bars mark the approximate position of long period comets on Fig. 9. The higher point treats carbon as an atmophile; the lower point is for nitrogen alone. The mass distribution of comets is quite uncertain. Long period comets are assigned the nominal value



FIG. 8. The parameter F_{χ} , the fraction of impactors striking Ganymede, Callisto, and Titan with (a) impact velocities less than v_c (top curve in Fig. 7), as calculated for Uranus–Neptune planetesimals; (b) impact velocities less than v_c^* (bottom curve in Fig. 7), also for U–N planetesimals; and (c) impact velocities less than v_c for Oort cloud comets.

q = 1.71. Short period comets, which may derive from and which provide the principle evidence for a Kuiper belt (Duncan *et al.* 1988), appear to be described by a lower value of $q \approx 1.5$ (e.g., Donnison 1986). The possible connection between the hypothetical Kuiper belt and short period comets is obviously germane to our discussion. If this lower value of q is used, it is much easier for Titan to accumulate an atmosphere, and an atmosphere for Callisto—particularly if carbon-based—becomes a distinct possibility.

The differences between the three satellites are largest for small values of q, and vanish as $q \rightarrow 2$. The differences

are large at small q because the primary source of the atmosphere is the occasional large impact that hits slowly enough not to blow itself into space. The relative frequency of such slow impacts is proportional to F_{χ} . The convergence of the models as $q \rightarrow 2$ was alluded to earlier in the discussion concerning Fig. 4: as $q \rightarrow 2$, the major source of the atmosphere shifts from large impactors to impactors too small to cause atmospheric cratering. Unlike the large impactors, which supply volatiles only if they hit at low velocity, the small impactors all contribute their atmosphere to the atmosphere regardless of impact velocity. Thus as $q \rightarrow 2$ the source becomes independent

	v_c given by Eq. (54)		v_c^* given by Eq. (55)			
	h = 0.33	h = 0.5	h = 0.8	h=0.33	h = 0.5	h = 0.8
	<u> </u>	Uranu	s-Neptune planetesin	nals		
Ganymede	0.028	0.0029	0	0.0004	0	0
Callisto	0.12	0.039	0.0076	0.015	0.0028	0.0001
Titan	0.79	0.37	0.11	0.22	0.073	0.016
			Oort cloud comets			
Ganymede	0.016	0.0042	0.0005	0.002	0.0002	0
Callisto	0.029	0.0084	0.0015	0.003	0.0005	0
Titan	0.21	0.081	0.021	0.045	0.013	0.0023

TABLE IVSome Representative Values of F_{χ}

of F_{χ} , and the differences between Callisto and Titan disappear as well.

5.6. Two Sensitivity Tests

Until now, we have assumed that when atmospheric cratering takes place, the impactor also escapes, leaving none of itself behind. Thus an efficient atmospheric cratering regime ($F_{\eta} \approx 1$) is at the same time a poor accumulative regime ($F_{\chi} \ll 1$), and vice versa. Both effects contribute to the potential instability of a coaccreting atmosphere; it is not obvious a priori which is more important. Nor, for



FIG. 9. Like Figs. 4 and 5, but with impactor atmophile content y as the dependent variable. The different curves, which are labeled by the parameter h, use values of F_{χ} appropriate to Titan and Callisto. Atmospheres accumulate above the curves. The most optimistic assumption (h = 1/3) for Callisto roughly coincides with the most pessimistic (h = 4/5) for Titan. The expansive error bars mark the approximate position of comets on this plot. The higher point treats carbon as an atmophile; the lower point is for nitrogen alone.

that matter, is it entirely obvious that all the material deriving from the impactor necessarily escapes in an atmospheric cratering event. It is for this reason, primarily, that we have formally retained separate identities for F_{χ} and F_{η} ; now we will exploit this.

An interesting sensitivity test that isolates the effect of atmospheric cratering is to assume that all the volatiles in all the impactors are retained, whether or not atmospheric erosion takes place. This can be done relatively simply, by arbitrarily setting $F_{\chi} = 1$ while leaving F_{η} adjustable. As an assumption this is at least as extreme as complete escape in atmospheric cratering events, but it is analytically tractable. With $F_{\chi} = 1$, Eq. (47) for the critical values of F_{η} and y reduces to

$$F_{\eta}(2-q)\left\{\xi_{0}\left(\frac{3-q}{2q-2}\right)\right\}^{2-q} = y^{q-1}.$$
 (57)

The consequences of this equation are displayed in Fig. 10. The figure was prepared specifically for Titan; comparable figures for Ganymede and Callisto are essentially identical. To facilitate comparison with our standard model (Fig. 4), we have plotted the quantity $1 - F_n$ rather than the critical value of F_{η} itself. In both figures, an atmosphere accumulates if the parameters q and $1 - F_{\eta}$ $(=F_{y}$ in Fig. 4) describing the velocity and mass distributions of the impactors plot above the curve labeled with the relevant value of y; i.e., atmospheres accumulate above the curves. Figure 10 shows that, no matter how efficiently impacting volatiles are retained, unless the impactors are atmophile-rich or q low, impact erosion prevents the accumulation of an atmosphere. For Callisto and Ganymede, where it is very likely that $1 - F_n \ll 1$, this means that for $q \approx 1.7$ (the present best estimate for comets), the impactors would still need to have y > 0.01for an atmosphere to accumulate, even with the most



optimistic assumption regarding atmophile retention. Reality probably lies somewhere between the results shown in Figs. 4 and 10. We must conclude, somewhat paradoxically, that accretion of a late volatile-rich veneer of outer Solar System material would have tended to prevent Ganymede and Callisto accumulating atmospheres, or retaining any early atmospheres they may once have had.

However, this is not the whole story for Ganymede and Callisto. We showed in Fig. 2 that *during* accretion of a late veneer a thin atmosphere is expected. This thin atmosphere is the equilibrium between atmophile expulsion and delivery, and vanishes as the impact flux declines. Its thickness for a given point in accretion can be calculated as a function of f_{χ} and y using Eq. (35) for q = 3/2 or Eq. (46) otherwise. It is predicted to be thicker for Callisto than for Ganymede.

A second test is to ask how sensitive our results are to the tangent-plane approximation. In the tangent-plane approximation the escaping gas is mainly that within a few degrees of the horizon. Escape therefore depends on the assumption that the ejecta follow radial or descending trajectories. If on the other hand the trajectories tend to refract upward, as they do for shocked gas in the case of the massless point explosion (Ahrens and O'Keefe 1987), much of the atmosphere near the horizon might not be swept to space, and the tangent-plane approximation would greatly overestimate the amount of air expelled by impacts.

An alternative view of atmospheric cratering, proposed by Ahrens and O'Keefe (1987), treats the impact as a massless point explosion at the base of an exponential atmosphere. The proposed analogy to a bomb has some drawbacks, the most important of which is that impacts are anything but massless (the snowplow analogy used in the tangent-plane approximation can be considered its antithesis), but a result from that model may be useful in placing a lower bound on escape. Ahrens and O'Keefe find that, as with the tangent-plane model, there is a maximum amount of atmosphere that can escape in an explosion, but this maximum is much less than the atmosphere above the horizon. In particular, they state that a strong explosion can remove at most $\sim 10^{-5}$ of Earth's present atmosphere, only about 1/60th of the atmosphere above the tangent plane. Ahrens and O'Keefe employ a semianalytical model in which the relevant expressions are cast in terms of a dimensionless distance normalized to the scale height. Because the mass of air that escapes is proportional to the cross-section of the escaping volume, it follows that escape scales as H^2 , and therefore that ξ_0 can be approximated by

$$\xi_0 \approx \frac{6H^2}{R^2}.$$
 (58)

For Titan, this is about 9% of the atmosphere above the horizon. Lest there be any confusion, it should be emphasized that we are not here attempting to fully implement the AOK algorithm. The latter requires expressing an impact as an equivalent bomb, which, if possible, is not straightforward. Instead we have only used AOK to provide an alternative estimate for the maximum amount of air that can be expelled by an impact. Because this is less than in the tangent-plane approximation, the mass of the threshold impactor to effect erosion should be scaled down proportionately. Therefore we can continue to take $m_c = \xi_0 M Y_a$, which leaves the relevant equations developed in the previous sections changed only by the value of ξ_0 .

In Fig. 11 we compare thresholds for the accumulation of an atmosphere in the tangent-plane model to the corresponding thresholds implied by Eq. (58). The figure is analogous to Figs. 4 and 10. Obviously the lower rate of impact erosion improves prospects for accumulating an atmosphere, particularly for smaller values of q and larger values of y, but for our nominal choice of $q \approx 1.7$ the differences are not so large as to change things qualitatively: it is still difficult for Callisto to get an atmosphere, and well-nigh impossible for Ganymede to get one.

6. SOME ADDITIONAL OBSERVATIONS

For the analytical treatment developed here it is implicitly assumed that impacts are numerous enough to justify





FIG. 11. Another sensitivity test, also to be compared to Fig. 4. Here it is assumed that the maximum loss of atmosphere in an impact is about $6H^2/R^2$ of the atmosphere above the tangent-plane. This upper limit is consistent with the upper limit for atmospheric escape calculated by Ahrens and O'Keefe (1987) for a massless point explosion.

treating the mass and velocity distributions of the impactors as continuous functions. Similarly, any variations in impactor volatile content are assumed to average out. Inevitable stochastic fluctuations in these parameters are ignored. These should be good approximations for impactor distributions in which small objects are well represented by mass (i.e., those distributions with q > 1.8), since for these distributions numerous small impacts control both the loss and the supply of atmophiles. But for smaller values of q infrequent large impactors are the main source of volatiles. Stochastic fluctuations on the supply side can then be important and, when so, the effect is not necessarily symmetrical. In particular, it is imaginable that a late, low velocity impact of a large, volatile-rich object could unjustly parachute a planet into the invulnerable region of Fig. 2a. This scenario is unlikely for Titan, where the atmosphere is so enormous that luck would need have been embodied as an object bigger than \sim 3 \times 10²³ grams, nor has chance availed Callisto or Ganymede, but it is a possibility for Mars. Monte Carlo simulations should help clarify the role of luck in determining the present distribution of planets and atmospheres.

The atmospheres of Triton and Pluto appear to have the same basic composition—mainly N_2 with some CH_4 —as Titan's. This is a mild surprise in the standard model, since Titan's history is thought to differ radically from Triton's or Pluto's. In the standard model Titan accumu-

lated from material that condensed in the Saturnian subnebula, while Pluto and Triton accumulated directly from solar nebular condensates. Systematic bulk compositional differences are expected and observed: Triton and Pluto, both of which have densities of ~ 2.1 g/cm³, are denser than Titan, Ganymede, and Callisto, which have densities of ~ 1.9 g/cm³. The compositional difference is actually bigger than this, owing to the presence of ice II in the larger bodies. But these systematic compositional differences are not reflected in their atmospheres.

According to the arguments developed in this paper, Triton and Pluto might both be expected to have atmospheres. Because they are both thought to have accumulated in free space at distances comparable to the U-N planetesimals, neither would have been subject to the systematically high impact velocities seen by the Galilean satellites. Even after capture, Triton saw significantly lower impact velocities than Titan, since it has a lower orbital velocity (4.4 km/sec vs 5.6 km/sec) and its impactors smaller v_{∞} . This argument is compromised by Triton's low escape velocity ($v_{esc} = 1.45$ km/sec, Stone and Miner 1989): impact velocities on Triton are low enough that the presumption of substantial impact vaporization of water is probably violated for impacts that would otherwise be expected to expel atmosphere. Discussion of Triton is also complicated by the tendency of nitrogen to condense at this distance (it is much easier to retain condensed volatiles), and by the doubtful relevance of U-N planetesimals to post-capture Tritonian impacts. A more straightforward argument is that, as alluded to earlier, Triton and Pluto may themselves be representive of the veneering objects that gave Titan its atmosphere, in which case the composition of their atmospheres is axiomatic.

Another hint of an impact origin of Titan's atmosphere is that the D/H ratio on Titan is similar to that of Earth. Mars, carbonaceous meteorites, and Halley (Owen et al. 1986, de Bergh et al. 1989, Coustenis et al. 1989). In the standard model methane forms from reaction of subnebular hydrogen with CO, a reaction sped by the relatively high temperatures and pressures expected in the subnebular disk. According to Prinn and Fegley (1989), methane formation in the Jovian subnebula occurs at a pressure of order 1 bar and at a quench temperature of \sim 800 K; the required conditions in the Saturnian subnebula would not be very different. The high temperature ensures that subnebular methane acquires a roughly cosmic D/H ratio, which it should still have on Titan (and does have on Jupiter and Saturn). In the veneer model Titanian methane would be expected to have the composition of the impactors, as observed.

It is conceivable that Titanian methane could acquire the D/H ratio of water ice by reactions in Titan (D. J. Stevenson, personal communication). Such might be the expected consequence of mixing impact-generated plumes of hot water vapor with atmospheric or oceanic methane. This does not necessarily compromise the late veneer hypothesis, because in any event Titan's outermost 30 km or so would be exogenic if the late veneer is to supply the atmospheric nitrogen inventory. Crustal cometary ice presumably would also exhibit the Urano-Neptunian D/H signature. If on the other hand atmospheric D/H has equilibrated with deep, innately Titanian ice, the implication would be that Titan is fundamentally nonsolar. This would bode ill for the conventional warm subnebula, since the high temperatures and presures needed to synthesize methane from carbon monoxide demand that subnebular water acquire cosmic D/H. A cold subnebula-e.g., one in which water ice is never vaporized-would not be ruled out, but a cold subnebula would not offer a plausible site for methane or ammonia synthesis. Also, a cold subnebula is rather subversive, since the existence of the subnebula presumes that the solids in the regular satellite system condensed from the subnebula; there is no call for a subnebula if the solids are imported intact from outside.

The apparent absence of carbon monoxide atmospheres in the modern Solar System poses something of a problem. Of course this problem is not unique to the veneer model for Titan's atmosphere, since it is an apparently inevitable result of high speed impacts (which are not banished in the standard model), yet a problem it remains. We cannot here resolve it. But we remind the reader that carbon monoxide is not an especially stable substance. As a major atmospheric constituent it is at best metastable. It is subject to photochemical destruction, and its disproportionation under pressure or at low temperatures is prevented only by kinetic barriers. The main reason that CO is sometimes considered a potential major atmospheric constituent in the modern Solar System is that the key reaction (or reactions) that destroy it have yet to be identified. In our opinion this is but a matter of time.

7. CONCLUSIONS

The present distribution of atmospheres between Ganymede, Callisto, and Titan can be explained by the competition between impact erosion and impact supply of atmophile-rich late veneers. Lower impact velocities on Titan, due in part to Saturn sitting less deeply in the Sun's gravitational well and in larger part to Saturn's being much less massive than Jupiter, allowed Titan to accumulate an atmosphere while its Jovian cousins Callisto and Ganymede remained barren. Our model requires that the threshold impact velocity for atmospheric cratering by icy impactors be in the range of 10–14 km/sec. Although this velocity range is somewhat higher than that predicted by Melosh and Vickery's criterion, it agrees reasonably well with the implications of Schmidt-Housen crater energyscaling for nonporous targets. The present mass of Titan's atmosphere falls within the acceptable range of veneers, although tending perhaps to the high side. An intrinsic nitrogen abundance on the order of 1% (or more) appears to be required of the impactors. Owing to relatively low impact velocities on Titan, the most agreeable candidate objects for the proposed veneer are Uranus-Neptune planetesimals or (hypothetical) Kuiper belt comets. This version of the late volatile-rich veneer model naturally accounts for the observation that Titan's D/H ratio is similar to those of Earth, meteorites, and comets, and dissimilar to those of Jupiter and Saturn. It also accounts for the compositional similarity of Titan's atmosphere to Triton's and Pluto's, despite their radically different histories and bulk compositions.

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